KMUTT FIXED POINT
LABORATORY GROUP WORK

> BOOKLET

7 12 14 MAR 2024

Organized by KMUTT Fixed Point Theory And Application Research Group, Faculty of Science, King Mongkut's University of Technology Thonburi, Thailand

ABSTRACT

The Fixed-Point Research Laboratory of King Mongkut's University of Technology Thonburi usually organizes a group-based seminar Presentations among PhD and MSc students each semester. The purpose of the seminars is to discuss research problems and ideas among graduate students of the Lab and researchers outside the Lab. In this regard, it is a great pleasure to present you with the details of the presentation scheduled for Semester 2 of 2023. The details comprise the title of each presentation, the name of the presenters, their emails, and the abstract of the talks. This program is designed to be held online and onsite within March 2024.

PRESENTATION SCHEDULE

March 7, 2024

1.00 - 1.30 PM **Muhammad Ramzan and Muhammad Arif**
Heat Transfer Analysis of Radiator Using Different Shaped Nanoparticles Water-based Ternary Hybrid Nanofluids with Applications: A fractional Model

1.30 - 2.00 PM **Saqib Murtaza and Dolat Khan**
Performance Analysis of Non-Linear Couple Stress Ternary Nanofluid in a Channel

2.00 - 2.30 PM **Saowalak Jitngam and Yutthakan Chummongkhon**
The Local Rings Approve a PST Occuring in its Unitary Cayley Graph

March 12, 2024

1.00 - 1.30 PM **Abdulwahab Ahmad and Ibrahim Arzuka**
Golden ratio algorithms with new stepsize rules for variational inequlities

1.30 - 2.00 PM **Muhammad Saqlain and Urairat Deepan**
Improving Crop Production: A Comparative Study of Hypersoft Set Theory and Bi-Level Optimization for Decision-Making

2.00 - 2.30 PM **Sani Salisu and Wiparat Worapitpong**
On Minimization and Fixed Point Problems in Hadamard Spaces

PRESENTATION SCHEDULE

March 14, 2024

1.00 - 1.30 PM	**Jamilu Yahaya and Nasiru Salihu** *Efficient Nonlinear Conjugate Gradient Techniques for Vector Optimization Problems*
1.30 - 2.00 PM	**Natthaya Boonyam and Petcharaporn Yodjai** *Reflected-Proximal Method for Equilibrium Problems*
2.00 - 2.30 PM	**Adamu Inuwa, Mahmoud M. Yahaya, and Solomon Gebregiorgis** *Dynamical System for Solving Bilevel Equilibrium Problems*

2^{nd} **Semester, 2023**

Fixed Point Laboratory Group Work Series,
Fixed Point Theory and Applications Research Group, Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Thailand

Heat Transfer Analysis of Radiator Using Different Shaped Nanoparticles Water-based Ternary Hybrid Nanofluids with Applications: A fractional Model

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Abstract

The suspension of nanoparticles in conventional base fluids getting more attention from scholars and researchers due to its unique thermal performance in different fields of engineering sciences. Nanofluid performed well and showed satisfactory results in the heat transport phenomena which attracted the scientists to suspend different combinations of nanoparticles which are named "hybrid nanofluid". From the experimental investigations, it is found that the rate of heat transfer is higher for hybrid nanofluid as compared to unitary nanofluid. Based on the above motivation the present study is focused on considering water-based ternary hybrid nanofluid with three different shaped nanoparticles i.e., spherical-shaped aluminum oxide, cylindrical carbon nanotubes, and platelet-shaped for the advanced cooling process of the radiator. From the present analysis, it is found that this advanced water-based ternary hybrid nanofluid showed promising enhancement in the heat transfer rate as compared to hybrid and unitary nanofluid. The present problem is formulated in the form of momentum and energy equations in terms of partial differential equations along with physical initial and boundary conditions. Furthermore, we have considered water-based ternary hybrid nanofluid with different shaped nanoparticles in channel. For the exact solutions the Laplace and Fourier transforms are applied. The influence of all the flow parameters is highlighted using the computational software MATHCAD. Using water-based ternary hybrid nanofluid enhances the rate of heat transfer up-to 33.67% which shows a promising thermal performance in the heat transfer rate. Furthermore, we have used nanoparticles in different ratios and found some interesting results which can be applied in different engineering problems specially, in cooling process.

Keywords: Ternary hybrid nanofluid, channel flow, thermal performance, Different shaped of nanoparticles, Exact Solutions, Laplace and Fourier Transforms.

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2^{nd} **Semester, 2023**

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Performance Analysis of Non-Linear Couple Stress Ternary Nanofluid in a Channel

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Abstract

Nanofluids have enhanced thermo-physical properties compared to traditional fluids, positioning them as the promising successors in fluid technology. The use of nanofluids enables the attainment of optimal thermal efficiency via the incorporation of a minimal concentration of nanoparticles that are stably suspended inside conventional fluids. The use of nanofluids in engineering and industry is seeing a steady increase as a result of effective implementation. The enhanced thermophysical properties of nanofluids have a substantial impact on their efficacy in convection phenomena. The technology doesn't stop there; binary and ternary nanofluids are now being used to make regular fluids more efficient. Therefore, this paper aims to analyze the natural convection couple stress ternary nanofluid flow in a vertical channel. A homogeneous suspension of aluminum oxide, copper oxide, and titanium oxide nanoparticles is formed by dissolving tri-hybridized nanoparticles in a base fluid (water). The effects of pressure gradient and viscous dissipation is also considered in the analysis. By using the Fractal-fractional derivative operator with exponential kernel, the classical couple stress ternary nanofluid model has been generalized. The discretization of the generalized model was achieved by the implementation of the Crank-Nicolson technique, followed by the use of computer software to solve it. In order to investigate the characteristics of fluid flow and the dispersion of thermal energy within the fluid, numerical techniques were used to compute the solution, which was then graphed in relation to various physical factors. The graphical results illustrate that at a volume fraction of 0.04 (equivalent to 4% of the base fluid), the heat transfer rate of the ternary nanofluid flow exhibits a substantial increase compared to the binary and unary nanofluid flows. The increase in the rate of heat transmission results in the augmentation of thermophysical properties, including viscosity, thermal expansion, and heat capacity, among others, of the base fluid.

Keywords: Ternary nanofluid; Couple stress fluid; Viscous dissipation; Fractal fractional derivative; Crank Nicolson scheme.

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The Local Rings Approve a PST Occuring in its Unitary Cayley Graph

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Abstract

In this work, we use eigenvalues and eigenvectors of unitary Cayley graphs over finite local rings and elementary linear algebra to characterize which local rings approve a Perfect State Transfer (PST) occurring in its unitary Cayley graph.

Keywords: Local rings; Perfect State Transfer; Unitary Cayley graphs; Linear algebra

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2^{nd} **Semester, 2023**

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On Minimization and Fixed Point Problems in Hadamard Spaces

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Abstract

In this paper, we incorporate proximal point algorithm with some convex combination techniques to approximate a minimizer and a common fixed point of a family of multi-valued mappings in Hadamard spaces. We prove Δ-convergence and strong convergence theorems for a convex lower semi-continuous function and multi-valued quasi-nonexpansivemappings. Furthermore, we apply our results to find mean, median and solve equilibrium problems in Hadamard spaces. Finally, we give two numerical examples, one in a non-Hilbert space with a non-convex functional and the other in a Hilbert space, to demonstrate the convergence of the proposed method.

Keywords: Common fixed points; Demiclosedness-type property, Hadamard space; Multi-valued quasi-nonexpansive mappings; Proximal point algorithm; Strong convergence; Δ-convergence

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Golden ratio algorithms with new stepsize rules for variational inequlities.

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Abstract
The variational inequality problem is known and has been studied as a central problem in nonlinear analysis, especially in optimization, control theory, game theory, among other fields. One of the typical approaches for solving monotone variational inequality problems is by using the golden ratio algorithm. In this work, to improve and extend this algorithm to a wider class of problems, the authors introduced two golden ratio algorithms for pseudomonotone and Lipschitz variational inequalities in finite dimentional Hilbert spaces. The special point is that, their proposed algorithms employ new step-size rules that are simpler than those in the original golden ratio algorithm. Under some standerd conditions, they established the convergence and the convergence rate of the proposed algorithms. They numerically illustrate their algorithms' efficiency in comparison to other algorithms.
Keywords: Golden ratio. Lipschitz contuinity· Proximal point algorithm· Pesudomonotone operator· Variational inequality

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Improving Crop Production: A Comparative Study of Hypersoft Set Theory and Bi-Level Optimization for Decision-Making

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Abstract

In the agricultural industry, crop production economics and uncertainty are closely related. Due to variables including shifting market pricing, rising input costs, and unexpected weather patterns, farmers confront several economic difficulties. Their decision-making processes, resource allocation, and overall profitability may all be greatly impacted by these uncertainties. Farmers frequently use risk management techniques to handle these complications, including crop diversification, insurance, and the adoption of cutting-edge technology. This paper aims to provide the decision-making method for the farmers to increase its crop production and profitability using its resources. To address the issue of this decision-making, a fuzzy hypersoft set has been used with MCDM technique known as MULTIMOORA, and also solved by using bi-level optimization. Since the suggested strategies are used in case studies when there are several attributes, more bifurcations, and multiple decision-makers. Farmers may more successfully traverse the problems they encounter, guaranteeing sustainable livelihoods, food security, and a healthy agricultural sector for the future, by comprehending and effectively managing economic risks and adopting mathematical decision-making techniques.

Keywords: Hypersoft set; Game theory; Decision-making; Bi-level optimization

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Efficient Nonlinear Conjugate Gradient Techniques for Vector Optimization Problems

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Abstract

Conjugate gradient techniques are known for their simplicity and minimal memory usage. However, it is known that in the vector optimization context, the Polak-Ribiére-Polyak (PRP) and Liu-Storey (LS) conjugate gradient (CG) techniques fail to satisfy the sufficient descent property using Wolfe line searches. In this work, we propose a variation of the PRP and LS CG techniques. These techniques exhibit the desirable property of sufficient descent without any line search. Under certain mild assumptions and employing strong Wolfe conditions, we investigate the global convergence properties of the proposed techniques. The global convergence analysis extends beyond convexity assumption on the objective functions. Additionally, we present numerical experiments and comparisons to demonstrate the implementation, efficiency, and robustness of the proposed techniques.

Keywords: Conjugate gradient method; Line search algorithm; Pareto-optimality; Vector optimization.

References

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Reflected-Proximal Method for Equilibrium Problems

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Abstract

In this article, a reflected-proximal method for solving equilibrium problem is proposed. For this method, weak and strong convergence results together with the rate of convergence are obtained under standard assumptions. The method requires only one computation of the proximal per iteration thereby making it less computationally expensive. Furthermore, the case when the feasible set is the set of fixed points of enriched nonexpansive mappings is considered for the reflected-proximal method and non-reflected proximal. Finally, substantial results for a variational inequality problem are obtained as consequences of the main results. The methods herein are quite effective even theoretically and also are implementable.

Keywords: Enriched nonexpansive mapping; Equilibrium problem; Fixed point; Monotone mapping; Reflected-proximal scheme; Variational inequality.

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Dynamical System for Solving Bilevel Equilibrium Problems

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Abstract

In this work, we study a bilevel equilibrium problem; we propose a new algorithm by using dynamical systems. We also proved that the trajectory of this dynamical system converges to a desired solution.

Keywords: Dynamical system; Bilevel equibrium problems; Strong monotonicity; Subdifferentiability

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